## 3 PHASE CIRCUITS

The single phase system is used for the operation of almost all the domestic and commercial applications like lamps, fans, electric irons , TV's , refrigerators , washing machines, computers etc..., But it has its own limitations in Generation Transmission , Distribution and industrial applications . Due to these variations in these parameters, the single phase system has been replaced by poly phase system, which is commonly used for generation, transmission and distribution of Electric

B. Balaji Reddy Associate Professor Power.

## PHASE AND PHASE DIFFERENCE

Phase means windings or circuits and each of them having an alternating voltage of the same magnitude and frequency.

The angular displacement between adjacent electrical quantity is called phase difference and depends on No. of phases. The phase difference is denoted by $\Phi$
$\therefore$ Phase Difference $(\Phi)=\left(360^{\circ}\right.$ electrical $) /($ No. of Phases $)$
However the aboverelation is not good enough for the two phase system. In two phase system the phase difference is $90^{\circ}$.

## PHASE SEQUENCE

In $3 \phi$ system there are three voltages having same magnitude and frequency displacing by an angle of 120 . These three voltages can attain their maximum value in a particular order.

The order or sequence in which the all these voltages attain their maximum value in the $3 \phi$ is called PHASE SEQUENCE.

Generally the three phases may be represented by numbers i.e. $1,2,3$ or by letters $\mathrm{a}, \mathrm{b}$, c or by colors RED, YELLOW, BLUE. In INDIA they are named as RYB.

The RYB or YBR or BRY is considered as appositive phase sequence where as RBY, BYR, YRB are considered as negative phase sequence.

The main significance concern regarding the usage of finding the phase sequence is:
$>$ The direction of rotation of $3 \phi$ Induction motors depends upon the phase sequence of $3 \phi$ supply. To reverse the direction of rotation, the phase sequence of supply given to motor has to be changed.
> The parallel operation of alternators and T/F s is possible if the phase sequence is known.

## ADVANTAGES OF $3 \phi$ SUPPLY OVER $1 \phi$ SUPPLY

$>$ A poly phase $3 \phi$ system requires less conductor material than the $1 \phi$ supply for same amount of power at same voltage level.
> For same size of machine, the poly phase machine gives higher output than $1 \phi$ machine.
$>$ Poly phase motor gives uniform torque, where as $1 \phi$ motor gives pulsating torque.
$>3 \phi$ induction motors are self starting motors but $1 \phi$ motors are not self starting.
$>$ For same power rating the P.F of $1 \phi$ motors is less than $3 \phi$ motors
$>$ The parallel operation of $3 \phi$ alternators or transformers id simple compared to $1 \phi$ alternators and transformers.

## CONNECTION OF THREE PHASES

Each coil of three phases has two terminals: one is starting terminal represented by 1 and other is ending terminal represented by 2 , as shown below.


Therefore it requires 6 terminal or conductors which make total system expensive. Hence the three phases are generally interconnected.

The general methods of interconnection are

* STAR or WYE CONNECTION.
* DELTA or MESH CONNECTION.


## (a) STAR OR WYE CONNECTION

If all similar terminals (i.e., starting ends or finishing ends) of three phase windings are connected to a common point as shown in fig, then the id is called star connection. This common point is called as star point or Neutral point.


Star Connection


Phasor diagram

From the star connection, for balanced supply:
The phase voltages are $\mathrm{V}_{\mathrm{RN}}=\mathrm{V}_{\mathrm{BN}}=\hat{V}_{\mathrm{YN}}=\mathrm{V}_{\mathrm{PH}}$
The line voltages are $\mathrm{V}_{\mathrm{RY}}=\mathrm{V}_{\mathrm{YB}}=\mathrm{V}_{\mathrm{BRR}}=\mathrm{V}_{\mathrm{LINE}}$
Where $V_{R Y}=$ Vector difference of $V_{R N}$ and $V_{Y N}=V_{R N}-V_{Y N}$
Similarly $\mathrm{V}_{\mathrm{YB}}=\mathrm{V}_{\mathrm{Y}^{-}}-\mathrm{V}_{\mathrm{B}}$ and $\mathrm{V}_{\mathrm{BR}}=\mathrm{V}_{\mathrm{B}}-\mathrm{V}$

## RELATION BETWEEN LINE VOLTAGE AND PHASE VOLTAGES

From the vector diagram the potential difference between R and Y phase is $\mathrm{V}_{\mathrm{RY}}$


Vector diagram
i.e.; $\mathrm{V}_{\mathrm{RY}}=\mathrm{V}_{\mathrm{RN}}-\mathrm{V}_{\mathrm{YN}}=\mathrm{V}_{\mathrm{RN}}+\left(-\mathrm{V}_{\mathrm{YN}}\right)$

The resultant voltage $\mathrm{V}_{\mathrm{RY}}=\sqrt{ } \mathrm{V}_{\mathrm{RN}}{ }^{2}+\left(-\mathrm{V}_{\mathrm{YN}}\right)^{2}-2 \mathrm{~V}_{\mathrm{RN}}\left(-\mathrm{V}_{\mathrm{YN}}\right) \operatorname{COS}$ (angle between $\mathrm{V}_{\mathrm{R}}$ $\& V_{Y}$ )
$\mathrm{V}_{\mathrm{RY}}=\sqrt{\mathrm{V}_{\mathrm{RN}}{ }^{2}+\mathrm{V}_{\mathrm{YN}}{ }^{2}+2 \mathrm{~V}_{\mathrm{RN}} \mathrm{V}_{\mathrm{YN}} \mathrm{COS} 60}$
$\mathrm{V}_{\mathrm{RY}}=\sqrt{\mathrm{V}_{\mathrm{PH}}{ }^{2}+\mathrm{V}_{\mathrm{PH}}{ }^{2}+2 \mathrm{~V}_{\mathrm{PH}} \mathrm{V}_{\mathrm{PH}}} * 0.5$
(since
$\mathrm{V}_{\mathrm{RN}}=\mathrm{V}_{\mathrm{BN}=} \mathrm{V}_{\mathrm{YN}=}=\mathrm{V}_{\mathrm{PH}}$ )
$\mathrm{V}_{\text {LINE }}=\sqrt{3} \mathrm{~V}_{\mathrm{PH}}$

$$
\mathrm{V}_{\mathrm{RY}}=\mathrm{V}_{\mathrm{LINE}}
$$

Hence the line voltage $=\sqrt{3}$ Phase voltage.

## 1. RELATION BETWEEN LINE AND PHASE CURRENTS

In the star connected system, each conductor or wdg is connected to separate phase, so the current flowing through the line and phase are same. i.e.; the current in phase R-phase is $I_{R}$, current in $Y$ phase is $I_{Y}$.

For balanced supply $\mathrm{I}_{\mathrm{R}}=\mathrm{I}_{\mathrm{Y}}=\mathrm{I}_{\mathrm{B}}=\mathrm{I}_{\mathrm{PH}}=\mathrm{I}_{\mathrm{L}}$
Line current $\left(\mathrm{I}_{\mathrm{L}}\right)=$ phase current $\left(\mathrm{I}_{\mathrm{PH}}\right)$

## 2. POWER EXPRESSION

If ' $\Phi$ ' is the phase difference or phase angle between the phase current ( $\mathrm{I}_{\mathrm{Ph}}$ ) and voltage ( $\mathrm{V}_{\mathrm{Ph}}$ ), then the expression for power per phase is:
$\mathrm{P}=\mathrm{V}_{\mathrm{Ph}} \mathrm{I}_{\mathrm{Ph}} \operatorname{Cos} \Phi$------- 1- $\Phi$
$\mathrm{P}=3$ (Power per phase) ------ 3- $\Phi$
$=3\left(\mathrm{~V}_{\mathrm{Ph}} \mathrm{I}_{\mathrm{Ph}} \operatorname{Cos} \Phi\right)$
But for star connection, $\mathrm{V}_{\mathrm{Ph}}=\mathrm{V}_{\mathrm{L}} / \sqrt{3} \quad$ and $\mathrm{I}_{\mathrm{Ph}}=\mathrm{I}_{\mathrm{L}}$
$\therefore$ Power $(\mathrm{p})=3\left(\mathrm{~V}_{\mathrm{L}} / \sqrt{3}\right) \mathrm{I}_{\mathrm{L}} \operatorname{Cos} \Phi$ watts for $3-\Phi$
Reactive power $(\mathrm{Q})=3$ ( Reactive Power per phase)

$$
=3\left(\mathrm{~V}_{\mathrm{Ph}} \mathrm{I}_{\mathrm{Ph}} \operatorname{Sin} \Phi\right)
$$

Reactive power $(Q)=3\left(V_{L} / \sqrt{3}\right) I_{L} \operatorname{Sin} \Phi$ var
$\mathrm{Q}=\sqrt{ } 3 \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \operatorname{Sin} \Phi \quad$ var

## DELTA OR MESH CONNECTION

If all dissimilar ends of the $3 \Phi$ windings are joined together to form a closed path or the three ways are joined in series to form a closed path, then this connection is called delta or mesh connection. i.e.; if the starting ends of one phase is joined to finishing end of other phase and so on as shown below fig:


Delta conneqtiow.sakshieducation.corfector Diagram

## (i).RELATION BETWEEN PHASE AND LINE VOLTAGES

In delta or mesh connection, there is no neutral point, so the potential difference between the two phases is called line voltage and it is also equal to phase voltage i.e;
$\mathrm{V}_{\mathrm{RY}}=\mathrm{V}_{\mathrm{RB}}=\mathrm{V}_{\mathrm{YB}}=\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{Ph}}$
Line voltage $\left(\mathrm{V}_{\mathrm{L}}\right)=$ Phase voltage $\left(\mathrm{V}_{\mathrm{Ph}}\right)$

## (ii).RELATION BETWEEN PHASE AND LINE CURRENT

From the above fig, the line current is vector difference phase current of two phases concern let $\mathrm{I}_{\mathrm{YR}}, \mathrm{I}_{\mathrm{RB}}$ and $\mathrm{I}_{\mathrm{BY}}$ are the phase currents i.e., the line current through R phase is
$\mathrm{I}_{\mathrm{R}}=\mathrm{I}_{\mathrm{YR}}-\mathrm{I}_{\mathrm{RB}}$

$$
=\mathrm{I}_{\mathrm{YR}}+\left(-\mathrm{I}_{\mathrm{RB}}\right)
$$

Similarly current through phase is $\mathrm{I}_{\mathrm{Y}}=\mathrm{I}_{\mathrm{BY}-}-\mathrm{I}_{\mathrm{YR}}$

$$
=I_{B Y}+\left(-\mathrm{I}_{\mathrm{YR}}\right)
$$

$$
\mathrm{I}_{\mathrm{B}}=\mathrm{I}_{\mathrm{RB}}-\mathrm{I}_{\mathrm{BY}}
$$

$$
=I_{\mathrm{RB}}+\left(-\mathrm{I}_{\mathrm{BY}}\right)
$$

The vector diagram representation of the currents of delta connection is shown in fig.


## Vector Diagram

From the vector diagram, the line current $\mathrm{I}_{\mathrm{R}}$

$$
\begin{aligned}
\mathrm{I}_{\mathrm{R}} & =\sqrt{\left.\mathrm{I}_{\mathrm{YR}}^{2}+\left(-\mathrm{I}_{\mathrm{RB}}\right)^{2}-2 \mathrm{I}_{\mathrm{YR}}\left(-\mathrm{I}_{\mathrm{RB}}\right) \text { COS (angle between } \mathrm{I}_{\mathrm{YR}} \& \mathrm{I}_{\mathrm{RB}}\right)} \\
& =\sqrt{\mathrm{I}_{\mathrm{YR}}{ }^{2}+\mathrm{I}_{\mathrm{RB}}^{2}+2 \mathrm{I}_{\mathrm{YR}} \mathrm{I}_{\mathrm{RB}} \mathrm{COS} 60} \\
& =\sqrt{\mathrm{I}_{\mathrm{PH}}^{2}+\mathrm{I}_{\mathrm{PH}}^{2}+2 \mathrm{I}_{\mathrm{PH}} \mathrm{IPH} * 0.5} \\
\mathrm{I}_{\mathrm{L}} & =\sqrt{3} \mathrm{I}_{\mathrm{PH}}
\end{aligned}
$$

i.e.; line current $=\sqrt{ } 3$ Phase current

## (iii) POWER EXPRESSION

We know that power per phase $=\mathrm{V}_{\mathrm{Ph}} \mathrm{I}_{\mathrm{Ph}} \operatorname{Cos} \Phi$
Total power for 3- $\Phi$ or Active power $(\mathrm{p})=3 \mathrm{~V}_{\mathrm{Ph}} \mathrm{I}_{\mathrm{Ph}} \operatorname{Cos} \Phi$

$$
\begin{array}{ll}
=3 V_{L}\left(\mathrm{I}_{\mathrm{L}} / \sqrt{ } 3\right) \cos \Phi & \because \mathrm{I}_{\mathrm{PH}}=\mathrm{I}_{\mathrm{L}} / \sqrt{ } 3 \\
=\sqrt{ } 3 \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \operatorname{Cos} \Phi & \because \mathrm{~V}_{\mathrm{Ph}}=\mathrm{V}_{\mathrm{L}}
\end{array}
$$

Reactive power $(\mathrm{Q})=3$ (Reactive Power per phase)

$$
=3\left(V_{\mathrm{Ph}} \mathrm{I}_{\mathrm{Ph}} \operatorname{Sin} \Phi\right)
$$

Reactive power $(\mathbb{Q})=3 \mathrm{~V}_{\mathrm{L}}\left(\mathrm{I}_{\mathrm{L}} / \sqrt{3}\right) \operatorname{Sin} \Phi$ var

$$
\mathrm{Q}=\sqrt{3} \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \operatorname{Sin} \Phi \quad \operatorname{var}
$$

## Difference between star and delta connection:

| Star Connection | Delta Connection |
| :--- | :--- |
| 1. Similar ends are joined together <br> i.e. all starting or finishing ends <br> are joined together to form a star <br> connection | 1. All dissimilar ends are joined <br> together or three windings are <br> joined in series to form a closed <br> path or delta connection. |
| 2. Phase Voltage= Line voltage $/ \sqrt{ } 3$ <br> phase current=Line current | 2. Phase current $=$ Line current <br> Phase voltage $=$ Line voltage |
| 3. Available of neutral point |  |$\quad$| 3. Neutral point is not available |
| :--- | :--- |

## Measurement of power in 3 - $\Phi$ system (Balanced or unbalanced system)

The power in 3- $\Phi$ system can be measured by using following methods

1. Three wattmeter method
2. Two wattmeter method
3. Single wattmeter method

## THREE WATTMETER METHOD

In this method, three wattcmeters are connected in each of three phases of load whether star or delta connected. The current coil of each wattmeter carries the current of one coil only and pressure coil measure the phase voltage of the phase as shown below in fig


Fig: Three wattmeter method - Star


Fig: Three wattmeter method- Delta

The total power in load is given by algebraic sum of the readings .Let $\mathrm{W}_{1}, \mathrm{~W}_{2}, \mathrm{~W}_{3}$ are the readings of wattemeters then the total power supplied to $3-\Phi$ load is $\mathrm{P}=$ $\mathrm{W}_{1}+\mathrm{W}_{2}+\mathrm{W}_{3}$.


## Phasor diagram

The Three wattmeter method is suitable for measurement of 3-Ф unbalanced power. Let us consider $\mathrm{I}_{\mathrm{R}}, \mathrm{I}_{\mathrm{Y}}, \mathrm{I}_{\mathrm{B}}$ are the currents of R, Y, B phases respectively which are nothing but phase and line currents. From circuit $V_{R N}, V_{B N}, V_{Y N}$ be the phase voltages $\left(\mathrm{V}_{\mathrm{PH}}\right)$ and $\mathrm{V}_{\mathrm{RY}}, \mathrm{V}_{\mathrm{YB}}, \mathrm{V}_{\mathrm{BR}}$ be the line voltages $\left(\mathrm{V}_{\mathrm{L}}\right)$.

Current through wattmeter 1 is $I_{R}$ and voltage across pressure coil of wattmeter 1 is $\mathrm{V}_{\mathrm{RN}}$ now reading in wattmeter 1 is
$\mathrm{W}_{1}=\mathrm{V}_{\mathrm{RN}} \mathrm{I}_{\mathrm{R}} \cos \Phi_{1}$
$\mathrm{W}_{1}=\mathrm{V}_{\mathrm{PH}} \mathrm{I}_{\mathrm{PH}} \cos \Phi_{1}$
Current through wattmeter 2 is $\mathrm{I}_{\mathrm{Y}}$ and voltage across pressure coil of wattmeter 1 is $\mathrm{V}_{\mathrm{YN}}$ now reading in wattmeter 2 is:
$\mathrm{W}_{2}=\mathrm{V}_{\mathrm{YN}} \mathrm{I}_{\mathrm{y}} \cos \Phi_{2}$
$\mathrm{W}_{2}=\mathrm{V}_{\mathrm{PH}} \mathrm{I}_{\mathrm{PH}} \cos \Phi_{2}$
Current through wattmeter 3 is $\mathrm{I}_{\mathrm{B}}$ and voltage across pressure coil of wattmeter 3 is $V_{\text {BN }}$ now reading in wattmeter 3 is
$\mathrm{W}_{3}=\mathrm{V}_{\mathrm{YN}} \mathrm{I}_{\mathrm{y}} \cos \Phi_{3}$
$\mathrm{W}_{3}=\mathrm{V}_{\mathrm{PH}} \mathrm{I}_{\mathrm{PH}} \cos \Phi_{3}$

Total power measured by three wattcmeters is $\mathrm{P}=\mathrm{W}_{1}+\mathrm{W}_{2}+\mathrm{W}_{3}$

$$
\mathrm{P}=\mathrm{V}_{\mathrm{PH}} \mathrm{I}_{\mathrm{PH}} \cos \Phi_{1}+\mathrm{V}_{\mathrm{PH}} \mathrm{I}_{\mathrm{PH}} \cos \Phi_{2}+\mathrm{V}_{\mathrm{PH}} \mathrm{I}_{\mathrm{PH}} \cos \Phi_{3}
$$

## Two Wattmeter method

The two wattmeter method is suitable for both balanced and unbalanced load. In this method, the current coils of two wattcmeters are inserted in any two Phases and pressure coils of each joined to third phase.


Two wattmeter method- Star


Two wattmeter method- Delta

The total power absorbed by the $3 \Phi$ balanced load is the sum of powers obtained by wattcmeters $\mathrm{W}_{1}$ and $\mathrm{W}_{2}$. When load is assumed as inductive load, the vector diagram for such a balanced star connected load is shown below


## Vector Diagram

Let $\mathrm{V}_{\mathrm{RN}}, \mathrm{V}_{\mathrm{BN}}, \mathrm{V}_{\mathrm{YN}}$ are the phase voltages and $\mathrm{I}_{\mathrm{R}}, \mathrm{I}_{\mathrm{Y}}, \mathrm{I}_{\mathrm{B}}$ are currents (phase or line ). Since load is inductive, the current lags their respective phase voltages by phase angle ( $\Phi$ ).

Let the current through wattmeter $\mathrm{w}_{1}=\mathrm{I}_{\mathrm{R}}$
Potential difference across pressure coil of wattmeter $\mathrm{w}_{1}=\mathrm{V}_{\mathrm{RB}}=\mathrm{V}_{\mathrm{RN}}-\mathrm{V}_{\mathrm{BN}}$
From vector diagram phase angle between $V_{R B}$ and $I_{R}$ is $30-\Phi$.
$\therefore$ Reading of wattmeter $\mathrm{W}_{1}=\mathrm{V}_{\mathrm{RB}} \mathrm{I}_{\mathrm{R}} \cos (30-\Phi)$

$$
\begin{equation*}
=\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos (30-\Phi) \tag{1}
\end{equation*}
$$

Similarly current through wattmeter $\mathrm{w}_{2}=\mathrm{I}_{\mathrm{Y}}$
Potential difference across pressure coil of wattmeter $2 \mathrm{~W}_{2}=\mathrm{V}_{\mathrm{YB}}$

$$
=\mathrm{V}_{\mathrm{Y}}-\mathrm{V}_{\mathrm{B}}
$$

The phase difference / angle between $\mathrm{V}_{\mathrm{YB}}$ and $\mathrm{I}_{\mathrm{Y}}$ is $30+\Phi$
$\therefore$ Reading of wattmeter $\mathrm{W}_{2}=\mathrm{V}_{\mathrm{YB}} \mathrm{I}_{\mathrm{Y}} \cos (30+\Phi)$

$$
\begin{equation*}
=V_{L} \mathrm{I}_{\mathrm{L}} \cos (30+\Phi) \tag{2}
\end{equation*}
$$

Total power $(\mathbf{P})=\mathbf{w}_{1}+\mathrm{w}_{\mathbf{2}}$

$$
\begin{align*}
= & V_{L} I_{L} \cos (30-\Phi)+V_{L} I_{L} \cos (30+\Phi) \\
P & =\sqrt{ } 3 V_{L} I_{L} \cos \Phi \text { watts. } \tag{3}
\end{align*}
$$

Hence the sum of two wattemeters gives the total power absorbed by the $3 \Phi$ load.

## Similarly to find Power factor

$$
\begin{align*}
& \mathrm{w}_{1}-\mathrm{w}_{2}=\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos (30-\Phi)+\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos (30+\Phi) \\
& \mathbf{w}_{\mathbf{1}-\mathbf{w}_{\mathbf{2}}}=\mathbf{V}_{\mathbf{L}} \mathbf{I}_{\mathbf{L}} \sin \boldsymbol{\Phi} \tag{4}
\end{align*}
$$

Dividing equation (3) by (4)

$$
\begin{aligned}
& \frac{\sqrt{3}(w 1-w 2)}{w 1+w 2}=\frac{\sqrt{3} \text { VL IL } \sin \Phi}{\sqrt{3} \text { VL IL } \cos \Phi} \\
& \sqrt{3}(w 1-w 2)
\end{aligned}
$$

Power factor is nothing but $\mathbf{C O S} \Phi=\mathbf{C O S} \pm\left(\tan ^{-\mathbf{1}} \frac{\sqrt{3}(w \mathbf{1}-w 2)}{w 1+w \mathbf{2}}\right)$

## REACTIVE POWER MEASUREMENT WITH TWO WATTMETER METHOD

We know that $\frac{\sqrt{3}(w 1-w 2)}{w 1+w 2}=\operatorname{Tan} \Phi$


## Power triangle

In the balanced condition, from above relations and power triangle, the reactive power is given by $\sqrt{3}$ times the difference of readings of wattemeters used.

$$
\text { Reactive power }=\sqrt{ } 3\left(\mathrm{~W}_{1}-\mathrm{W}_{2}\right) \text { var }
$$

We know the value of $\left(\mathrm{W}_{1}-\mathrm{W}_{2}\right)$ from eqn (4)

$$
\Rightarrow \text { Reactive power }=\sqrt{ } 3\left(V_{L} I_{L} \sin \Phi\right) \text { var }
$$

Variations in wattmeter readings in 2 wattmeter method due to power factor
We know that, for balanced inductive load
Reading of wattmeter 1 is $W_{1}=V_{L} \mathrm{I}_{\mathrm{L}} \cos (30-\Phi)$


From above equation, it is clear that readings of wattcmeters not only depend on load but also depends on its phase angle i.e.
i. When $\Phi=0^{\circ}$ i.e. power factor $=\cos \Phi=$ unity (resistive load)

Then $\mathrm{W}_{1}=\mathrm{W}_{2}=\operatorname{Cos} 30^{\circ}$
The readings of both wattcmeters are same.
ii. When $\Phi=60^{\circ}$ i.e power factor $=\cos \Phi=0.5 \mathrm{lag}$

Then $\mathrm{W}_{1}=\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \operatorname{Cos}\left(30^{\circ}-60^{\circ}\right)=\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \operatorname{Cos} 30^{\circ}$

$$
\mathrm{W}_{2}=\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \operatorname{Cos}\left(30^{\circ}+60^{\circ}\right)=0
$$

Hence wattmeters 1 only read power
iii. When $90>\Phi>60$ i.e $0.5>\operatorname{Cos} \Phi>0$

When phase angle is 60 to 90 , then the wattmeter $W_{1}$ readings are positive and readings of wattmeter $\mathrm{W}_{2}$ will be reversed. For getting the total power, the readings of $W_{2}$ is to be subtracted from that of $W_{1}$.
iv. When $\Phi=90^{\circ}$ i.e power factor $=0$

Then $W_{1}=V_{L} \mathrm{I}_{\mathrm{L}} \operatorname{Cos}\left(30^{\circ}-90^{\circ}\right)=\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \operatorname{Cos} 60^{\circ}$

$$
\mathrm{W}_{2}=\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \operatorname{Cos}\left(30^{\circ}+90^{\circ}\right)=-\mathrm{V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \operatorname{Sin} 30^{\circ}
$$

These two readings are equal in magnitude but opposite in sign
$\therefore$ Total power $=\mathrm{W}_{1}+\mathrm{W}_{2}=0$

## SINGLE WATTMETER METHOD

The single wattmeter method is used to measure the power of 3-Ф balanced system. Let $\mathrm{Z}_{\mathrm{R}}, \mathrm{Z}_{\mathrm{Y}}, \mathrm{Z}_{\mathrm{B}}$ are the impedances of $\mathrm{R}, \mathrm{Y}, \mathrm{B}$ phases. $\mathrm{I}_{\mathrm{R}}, \mathrm{I}_{\mathrm{Y}}, \mathrm{I}_{\mathrm{B}}$ are currents through $\mathrm{R}, \mathrm{Y}, \mathrm{B}$ phases respectivelyakshieducation.com
$\mathrm{V}_{\mathrm{RN}}, \mathrm{V}_{\mathrm{BN}}, \mathrm{V}_{\mathrm{YN}}$ be the phase voltages $\left(\mathrm{V}_{\mathrm{PH}}\right)$ and $\mathrm{V}_{\mathrm{RY}}, \mathrm{V}_{\mathrm{YB}}, \mathrm{V}_{\mathrm{BR}}$ be the line voltages $\left(\mathrm{V}_{\mathrm{L}}\right)$.


From above diagram, the current through wattmeter is $\mathrm{I}_{\mathrm{R}}$, voltage across pressure coil is $\mathrm{V}_{\mathrm{RN}}$. Now wattmeter reading is:

$$
W=V_{\mathrm{RN}} \mathrm{I}_{\mathrm{R}} \cos \Phi=\mathrm{V}_{\mathrm{PH}} \mathrm{I}_{\mathrm{PH}} \cos \Phi
$$

Total power $=3^{*} \mathrm{~V}_{\mathrm{PH}} \mathrm{I}_{\mathrm{PH}} \cos \Phi$

$$
=\sqrt{ } 3 \mathrm{~V}_{\mathrm{L}} \mathrm{I}_{\mathrm{L}} \cos \Phi \quad \because \mathrm{~V}_{\mathrm{L}}=\sqrt{ } 3 \mathrm{~V}_{\mathrm{PH}} \& \mathrm{I}_{\mathrm{L}}=\mathrm{I}_{\mathrm{PH}}
$$

## MEASUREMENT OF REACTIVE POWER IN SINGLE WATTMETER METHOD

The reactive power of $3 \Phi$ circuit can be measured using compensated wattmeter. The circuit diagram of $3 \Phi$ star connection with compensated wattmeter is shown below.


Let $Z_{R}, Z_{Y}, Z_{B}$ are the impedances of $R, Y, B$ phases. $I_{R}, I_{Y}, I_{B}$ are currents through $\mathrm{R}, \mathrm{Y}, \mathrm{B}$ phases respectively.
$\mathrm{V}_{\mathrm{RN}}, \mathrm{V}_{\mathrm{BN},}, \mathrm{V}_{\mathrm{YN}}$ be the phase voltages $\left(\mathrm{V}_{\mathrm{PH}}\right)$ and $\mathrm{V}_{\mathrm{RY}}, \mathrm{V}_{\mathrm{YB}}, \mathrm{V}_{\mathrm{BR}}$ be the line voltages $\left(V_{L}\right)$.

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{RN}=} \mathrm{V}_{\mathrm{BN}}=\mathrm{V}_{\mathrm{YN}=} \mathrm{V}_{\mathrm{PH}} \\
& \mathrm{~V}_{\mathrm{RY}=}=\mathrm{V}_{\mathrm{YB}}=\mathrm{V}_{\mathrm{BR}}=\mathrm{V}_{\mathrm{LINE}}
\end{aligned}
$$



## Vector diagram

Current through the current coil of wattmeter is $\mathrm{I}_{\mathrm{R}}$
Voltage across pressure coil of wattmeter $=\mathrm{V}_{\mathrm{YB}}=\mathrm{V}_{\mathrm{Y}}-\mathrm{V}_{\mathrm{B}}$
Wattmeter reading $=\sqrt{3} \mathrm{~V}_{\mathrm{Fw}}$ Www Sidkshieducation.com

$$
=\sqrt{3}\left(\sqrt{ } 3 \mathrm{~V}_{\mathrm{PH}} \mathrm{I}_{\mathrm{PH}} \operatorname{Sin} \Phi\right)
$$

$\mathrm{Q}=3 \mathrm{~V}_{\mathrm{PH}} \mathrm{I}_{\mathrm{PH}} \operatorname{Sin} \Phi$

## UNBALANCED SYSTEMS

A system is said to be balanced system if the impedances or phase angle or frequencies of three phases is same otherwise it is called as unbalanced system.

There are two types of unbalanced systems. Those are

1. Three phase four wire system (star connection with neutral)
2. Three phase three wire system(Delta or Star connection with Neutral)

## 1. THREE PHASE FOUR WIRE SYSTEM

The three phase three wire unbalanced system can be solved by any one of the following methods.
i) Star to Delta conversion method
ii) Loop or Mesh analysis method.
iii) Milliman's Method

## STAR TO DELTA CONNECTION

Star to Delta conversion method is used to solve $3 \Phi, 3$ wire unbalanced system. Let us consider the $3 \Phi$ star connection without neutral as shown below. Let the phase sequence be R, Y \& B.

Let $Z_{R}, Z_{Y}, Z_{B}$ are the impedances of $R, Y, B$ phases. $I_{R}, I_{Y}, I_{B}$ are currents through R, Y, B phases respectively.
$\mathrm{V}_{\mathrm{RN}}, \mathrm{V}_{\mathrm{BN}}, \mathrm{V}_{\mathrm{YN}}$ be the phase voltages $\left(\mathrm{V}_{\mathrm{PH}}\right)$ and $\mathrm{V}_{\mathrm{RY}}, \mathrm{V}_{\mathrm{YB}}, \mathrm{V}_{\mathrm{BR}}$ be the line voltages $\left(V_{L}\right)$.

$$
\mathrm{V}_{\mathrm{RN}} \neq \mathrm{V}_{\mathrm{BN}} \neq \mathrm{V}_{\mathrm{YN}} \neq \mathrm{V}_{\mathrm{PH}}
$$


$\mathrm{Z}_{\mathrm{RY}}, \mathrm{Z}_{\mathrm{RB}}$ and $\mathrm{Z}_{\mathrm{YB}}$ are the branch impedances and are determined as
$Z_{R Y}=Z_{R}+Z_{Y}+\left(Z_{R} Z_{Y}\right) / Z_{B}$
$\mathrm{Z}_{\mathrm{RB}}=\mathrm{Z}_{\mathrm{R}}+\mathrm{Z}_{\mathrm{B}}+\left(\mathrm{Z}_{\mathrm{R}} \mathrm{Z}_{\mathrm{B}}\right) / \mathrm{Z}_{\mathrm{Y}}$
$\mathrm{Z}_{\mathrm{YB}}=\mathrm{Z}_{\mathrm{Y}}+\mathrm{Z}_{\mathrm{B}}+\left(\mathrm{Z}_{\mathrm{Y}} \mathrm{Z}_{\mathrm{B}}\right) / \mathrm{Z}_{\mathrm{R}}$

## Brach Currents

If $\mathrm{I}_{\mathrm{RY}}, \mathrm{I}_{\mathrm{YB}} \mathrm{I}_{\mathrm{BR}}$ are the Brach currents, then can be determined as:

$$
\begin{gathered}
\mathrm{I}_{\mathrm{RY}}=\mathrm{V}_{\mathrm{RY}} \angle 0^{\circ} / \mathrm{Z}_{\mathrm{RY}} \\
\mathrm{I}_{\mathrm{YB}}=\mathrm{V}_{\mathrm{YB}} \angle-120^{\circ} / \mathrm{Z}_{\mathrm{RY}} \\
\mathrm{I}_{\mathrm{RB}}=\mathrm{V}_{\mathrm{RB}} \angle-240^{\circ} / \mathrm{Z}_{\mathrm{RY}}
\end{gathered}
$$

## Line currents

$\mathrm{I}_{\mathrm{R}}, \mathrm{I}_{\mathrm{Y}}, \mathrm{I}_{\mathrm{B}}$ are the line currents and can be determined as follows:

$$
\text { At point ' } a \text { ' } I_{R B}+I_{R}=I_{R Y}
$$

$$
\mathrm{I}_{\mathrm{R}}=\mathrm{I}_{\mathrm{RY}}-\mathrm{I}_{\mathrm{RB}}
$$

At point 'b' $\mathrm{I}_{\mathrm{YB}}+\mathrm{I}_{\mathrm{Y}}=\mathrm{I}_{\mathrm{RB}}$

$$
\mathrm{I}_{\mathrm{Y}}=\mathrm{I}_{\mathrm{RB}}-\mathrm{I}_{\mathrm{YB}}
$$

At point ' $c$ ' $I_{R Y}+I_{Y}=I_{Y B}$

$$
\mathrm{I}_{\mathrm{Y}}=\mathrm{I}_{\mathrm{YB}}-\mathrm{I}_{\mathrm{RY}}
$$

The voltage across $\mathrm{Z}_{\mathrm{R}}$ is $\mathrm{V}_{\mathrm{ZR}}=\mathrm{I}_{\mathrm{R}} \mathrm{Z}_{\mathrm{R}}$
Voltage across $Z_{Y}$ is $V_{Z Y}=I_{Y} Z_{Y}$
Voltage across $Z_{B}$ is $V_{Z B}=I_{B} Z_{B}$

## LOOP OR MESH ANALYSIS

The loop or mesh analysis method is used to solve the $3 \Phi$, star without neutral system. Let us consider a star without neutral as shown below. Let the phase sequence as RYB

Let $\mathrm{Z}_{\mathrm{R}}, \mathrm{Z}_{\mathrm{Y}}, \mathrm{Z}_{\mathrm{B}}$ are the impedances of R , $\mathrm{Y}, \mathrm{B}$ phases.

Voltage across $\mathrm{R} \& \mathrm{Y}$ is $\mathrm{V}_{\mathrm{RY}} \angle 0^{\circ}$
Voltage across $Y$ \& $B$ is $V_{Y B}<-120$


Voltage across $R \& B$ is $V_{R B}<-240^{\circ}$
$\mathrm{I}_{\mathrm{R}}, \mathrm{I}_{\mathrm{Y}}, \mathrm{I}_{\mathrm{B}}$ are the line or phase currents
Applying KVL to loop 1

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{RY}} \angle 0=\mathrm{I}_{1} \mathrm{Z}_{\mathrm{R}}+\left(\mathrm{I}_{1}-\mathrm{I}_{2}\right) \mathrm{Z}_{\mathrm{Y}} \\
& \mathrm{~V}_{\mathrm{RY}} \angle 0=\mathrm{I}_{1} \mathrm{Z}_{\mathrm{R}}+\mathrm{I}_{1} \mathrm{Z}_{\mathrm{Y}}-\mathrm{I}_{2} \mathrm{Z}_{\mathrm{Y}} \\
& \mathrm{~V}_{\mathrm{RY}} \angle 0=\mathrm{I}_{1}\left(\mathrm{Z}_{\mathrm{R}}+\mathrm{Z}_{\mathrm{Y}}\right)-\mathrm{I}_{2} \mathrm{Z}_{\mathrm{Y}}
\end{aligned}
$$

Find $\mathrm{I}_{1}$ from above equation

$$
\mathrm{I}_{1}=\frac{\mathrm{V}_{\mathrm{RY}} \angle \mathbf{0}+\mathrm{ZYI}_{2}}{\mathrm{Z}_{\mathrm{R}}+\mathrm{Z}_{\mathrm{Y}}}
$$

$$
\begin{gathered}
\mathrm{V}_{\mathrm{YB}} \angle-120=\left(\mathrm{I}_{2}-\mathrm{I}_{1}\right) \mathrm{Z}_{\mathrm{Y}}+\mathrm{Z}_{\mathrm{B}} \mathrm{I}_{2} \\
\mathrm{~V}_{\mathrm{YB}} \angle-120=-\mathrm{I}_{1} \mathrm{Z}_{\mathrm{Y}}+\left(\mathrm{Z}_{\mathrm{B}}+\mathrm{Z}_{\mathrm{Y}}\right) \mathrm{I}_{2}
\end{gathered}
$$

Now by substituting I1 in above equation we get $\mathrm{I}_{2}$
From circuit branch currents are $\mathrm{I}_{\mathrm{R}}=\mathrm{I}_{1}$

$$
\begin{gathered}
\mathrm{I}_{\mathrm{Y}}=\mathrm{I}_{1}-\mathrm{I}_{2} \\
\mathrm{I}_{\mathrm{B}}=-\mathrm{I}_{2}
\end{gathered}
$$

## MILLIMAN'S THEOREM

Consider a $3 \Phi$ star without neutral is excited by star connected supply as shown in fig.

Let $\mathrm{Z}_{\mathrm{R}}, \mathrm{Z}_{\mathrm{Y}}, \mathrm{Z}_{\mathrm{B}}$ are the impedances of $\mathrm{R}, \mathrm{Y}, \mathrm{B}$ phases. $\mathrm{I}_{\mathrm{R}}, \mathrm{I}_{\mathrm{Y}}, \mathrm{I}_{\mathrm{B}}$ are the currents of $\mathrm{R}, \mathrm{Y}, \mathrm{B}$ phases.


According to millimans theorem,
The voltage at load star point o' w.r.t source point o is Voo is given as follows:

$$
\mathrm{V}_{\infty}=\frac{\frac{\mathrm{V}_{\mathrm{RO}} \angle 0}{\mathrm{Z}_{\mathrm{R}}}+\frac{\mathrm{V}_{\mathrm{YO}} \angle-120}{\mathrm{Z}_{\mathrm{Y}}}+\frac{\mathrm{V}_{\mathrm{BO}} \angle-240}{\mathrm{Z}_{\mathrm{B}}}}{\frac{1}{\mathrm{Z}_{\mathrm{R}}}+\frac{1}{\mathrm{ZY}_{\mathrm{Y}}}+\frac{1}{\mathrm{Z}_{\mathrm{B}}}}
$$

Voltage across $\mathrm{Z}_{\mathrm{R}}$ of load is $\mathrm{V}_{\mathrm{RO}}{ }^{\prime}=\mathrm{V}_{\mathrm{OO}}{ }^{\prime}-\mathrm{V}_{\mathrm{RO}}$

Voltage across $\mathrm{Z}_{\mathrm{Y}}$ of load is $\mathrm{V}_{\mathrm{YO}}{ }^{\prime}=\mathrm{V}_{\mathrm{OO}},-\mathrm{V}_{\mathrm{YO}}$
Voltage across $\mathrm{Z}_{\mathrm{B}}$ of load is $\mathrm{V}_{\mathrm{BO}}{ }^{\prime}=\mathrm{V}_{\mathrm{OO}}{ }^{\prime}-\mathrm{V}_{\mathrm{BO}}$

Branch currents are $\mathrm{I}_{\mathrm{R}}=\mathrm{V}_{\mathrm{RO}} / \mathrm{Z}_{\mathrm{R}}=\left(\mathrm{V}_{\mathrm{OO}},-\mathrm{V}_{\mathrm{RO}}\right) / \mathrm{Z}_{\mathrm{R}}$

$$
\begin{aligned}
& \mathrm{I}_{\mathrm{Y}}=\mathrm{V}_{\mathrm{YO}}{ }^{\prime} / \mathrm{Z}_{\mathrm{Y}}=\left(\mathrm{V}_{\mathrm{OO}}{ }^{\prime}-\mathrm{V}_{\mathrm{YO}}\right) / \mathrm{Z}_{\mathrm{Y}} \\
& \mathrm{I}_{\mathrm{B}}=\mathrm{V}_{\mathrm{BO}} / \mathrm{Z}_{\mathrm{B}}=\left(\mathrm{V}_{\mathrm{OO}},-\mathrm{V}_{\mathrm{BO}}\right) / \mathrm{Z}_{\mathrm{B}}
\end{aligned}
$$

